

# Local cubic vertex functions for three massless higher even spin fields on spaces $AdS_D$ : An analytic approach

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## Abstract

Local cubic vertex functions of three higher even spin fields on  $AdS_D$  are constructed from the Green function of three conserved currents that are dual to the higher spin fields. Conservation of the currents implies lowest order gauge invariance. These vertex functions appear by the  $UV$  divergence as the residue of the highest order pole in the dimensional regularization parameter  $\epsilon$ . In fact  $N$ -point Green functions of such conserved currents produce a series of poles up to the order  $N - 1$ . The method works for even  $D$  and maintains covariance at any step. The resulting formula is quite concise.

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# 1 Introduction

It is the aim of this article to construct vertex functions for three massless higher spin fields of even spin on  $AdS$  spaces. This construction is done in such fashion that these vertex functions are local and observe gauge invariance of lowest order. A higher spin field of a symmetric tensor representation of rank ("spin")  $s$  is contracted with a conserved current of the same representation. This current is built bilinearly from a real scalar conformal free field admitting only even rank tensors. The vacuum expectation value of three such currents is UV divergent, and this divergence can be characterized by a polynomial in  $\epsilon^{-1}$ , where this parameter  $\epsilon$  can be introduced by a deformation of the dimension  $D$  of the  $AdS$  space. For  $N$  such currents the  $N$ -point Green function yields a UV divergence described by a polynomial of degree  $N - 1$ . We are interested here only in the residue of the highest order pole term. It is known to preserve a symmetry such as gauge invariance.

We neglect trace terms from the beginning, so that our result holds for tracefree higher spin fields. All other terms are kept. This method has been developed for flat spaces [1], in which case we were also able to successfully compare the results with the general known formulas [8]. It seems that the postulate of  $AdS$  covariance introduces new problems. The reader will find, however, that the choice of an appropriate mathematical formalism simplifies the whole task dramatically. This formalism was developed in [2] a few years ago when the quantum one loop trace anomaly of the same fields and currents was analysed for  $AdS_4$ . The restriction to the dimension  $D = 4$  was motivated by the desire to compare the results with known anomalies in gravitation theory. In the present context such motivation does not seem to exist and we can as well and will deal with cases of any even space dimension. The relevant formulae of [2] are generalized for this purpose to any even dimension  $D$ .

The history of cubic vertex functions on flat or (anti)deSitter spaces is long and has put forward different aspects of interest. First we must mention the seminal work of Fradkin and Vasiliev [3]. The frame-like approach to vertex functions of any order and non-abelian gauge symmetry proposed by these authors has developed in recent decades into several directions (for a recent source see [4]). We present instead a quantum field theoretic algorithm applicable to lowest order gauge invariance of any vertex function on  $AdS_D$ .

In flat spaces the general 3-vertex functions were constructed in [8] and restrictions on the form of these vertex function that were achieved earlier [7] could be verified. Connection with string theory was emphasized in [9]. Contrary to the very satisfactory results for flat spaces, the situation for  $AdS$  spaces is still not completely satisfactory at least in the sense that the resulting formulae do not exhibit such a simple and beautiful form as in the flat case. There are different methods applied to solve the  $AdS$  cases. Using flat ambient spaces and then reducing the dimension by one looks as a convincing ansatz but technical complications show up when this is done explicitly [5], [6]. Instead of this ansatz we use now a method based on

quantum field theory and the regularization techniques of UV divergences leading to a compact final formula.

## 2 Scalar fields and conserved currents

It is remarkable though of course natural that we go back to the analysis of anomalies of 2-loop functions on  $AdS_D$  in [2] when we study the singular part of N-loop functions. The scalar field and the currents are in fact the same. The free scalar conformal field  $\sigma(z)$  has the two point function

$$\langle \sigma(z_1) \sigma(z_2) \rangle = w(\zeta), \quad \zeta - 1 = \frac{(z_1 - z_2)^2}{2z_{1,0}z_{2,0}} \quad (2.1)$$

where Poincare coordinates are used. The function  $w(\zeta)$  is a Legendre function of the second kind which in terms of a Gaussian hypergeometric function is

$$w(\zeta) = \frac{\Gamma(\Delta)}{(2\pi)^{\frac{D}{2}}} \zeta^{-\Delta} F\left(\frac{\Delta}{2}, \frac{\Delta+1}{2}; \frac{1}{2}; \zeta^{-2}\right) \quad (2.2)$$

and the parameters  $\mu, \Delta$  denote (in the scalar field case)

$$\Delta = \frac{D}{2} - 1 \quad (2.3)$$

where  $\Delta$  is the conformal dimension of the scalar field  $\sigma$ . For arbitrary  $D$  the two-point function is identical with

$$w(\zeta) = \frac{\Gamma(\Delta)}{2(2\pi)^{\frac{D}{2}}} \left[ \frac{1}{(\zeta - 1)^\Delta} + \frac{1}{(\zeta + 1)^\Delta} \right] \quad (2.4)$$

that is rational for even  $D$ . For odd  $\Delta (D = 4n)$  replacing  $\zeta$  by  $-\zeta$  changes the sign of the function  $w(\zeta)$ . This symmetry is denoted *antipodal symmetry*, the sign change *antipodal parity*. We recognize that the 2-point function has two singular points at  $\zeta = +1$  and  $\zeta = -1$ , such pair is called an antipodal pair. Correspondingly a 3-point function of three currents has eight maximal singular points when

$$\zeta_{1,2} = \pm 1, \zeta_{2,3} = \pm 1, \zeta_{3,1} = \pm 1 \quad (2.5)$$

Analogously the 3-vertex consists of eight parts, seven of which are automatically generated from the eighth one by antipodal symmetry. We shall choose this one to belong to

$$\zeta_{1,2} = \zeta_{2,3} = \zeta_{3,1} = 1 \quad (2.6)$$

and concentrate our work on this alone.

For the conserved current we can take the expression  $J^{(s)}(z; a)$  from [2], equ. (1),  $s$  is even. The complete expression of  $J^{(s)}(z; a)$  is a polynomial in  $a^2$  of degree  $\frac{s}{2}$

and each factor of  $(a^2)^n$  is a polynomial in  $L^{-2}$  ( $L$  is the radius of the  $AdS$  space) of degree  $n$ . Since we want to consider only the traceless terms of the cubic vertex, we concentrate only on the  $(a^2)^0$  part

$$J^{(s)}(z; a) = 1/2 \sum_{p=0}^s A_p (a \nabla)^{s-p} \sigma(z) (a \nabla)^p \sigma(z) + \text{trace terms} \quad (2.7)$$

The coefficients are

$$A_p^{(s)} = (-1)^p \binom{s}{p} \frac{(D/2 - 2)!(s + D/2 - 2)!}{(p + D/2 - 2)!(s - p + D/2 - 2)!} \quad (2.8)$$

We are interested in the loop Green function

$$< J^{(s_1)}(z_1; a_1) \quad J^{(s_2)}(z_2; a_2) \quad J^{(s_3)}(z_3; a_3) > \quad (2.9)$$

This Green function is evaluated by Wick's theorem using (2.2).

### 3 Evaluating the Green function

Let  $F(\zeta)$  be an analytic function of  $\zeta$ . Then we can use the formulae [2], equs. (24),(25)

$$(a, \nabla_1)^p (b, \nabla_2)^q F(\zeta(z_1, z_2)) = \sum_{n=0}^{q-p} \frac{p!q!}{n!(q-n)!(p-q+n)!} \times I_{1,2}^{q-n} I_1(1, 2)^{p-q+n} I_2(1, 2)^n F^{(p+q)}(\zeta(z_1, z_2)) + \text{traces} \quad (3.1)$$

$$I_1(1, 2) = (a, \partial_1) \zeta(z_1, z_2) \quad (3.2)$$

$$I_2(1, 2) = (b, \partial_2) \zeta(z_1, z_2) \quad (3.3)$$

$$I_{1,2} = (a, \partial_1)(b, \partial_2) \zeta(z_1, z_2) \quad (3.4)$$

where

$$F^{(k)}(\zeta) = \frac{d^k}{(d\zeta)^k} F(\zeta) \quad (3.5)$$

For the Green function we get in this fashion (up to trace terms)

$$\sum_{p_1=0}^{s_1} \sum_{p_2=0}^{s_2} \sum_{p_3=0}^{s_3} A_{p_1}^{(s_1)} A_{p_2}^{(s_2)} A_{p_3}^{(s_3)} (a, \nabla_1)^{p_1} (b, \nabla_2)^{s_2-p_2} w(\zeta_{1,2}) \times (b, \nabla_2)^{p_2} (c, \nabla_3)^{s_3-p_3} w(\zeta_{2,3}) (c, \nabla_3)^{p_3} (a, \nabla_1)^{s_1-p_1} w(\zeta_{3,1}) \quad (3.6)$$

which by equs. (3.1) to (3.4) yields

$$\sum_{p_1=0}^{s_1} \sum_{p_2=0}^{s_2} \sum_{p_3=0}^{s_3} A_{p_1}^{(s_1)} A_{p_2}^{(s_2)} A_{p_3}^{(s_3)} \mathbf{Q}_{p_1, p_2, p_3}^{(s_1, s_2, s_3)} \times w^{(p_1+s_2-p_2)}(\zeta_{1,2}) w^{(p_2+s_3-p_3)}(\zeta_{2,3}) w^{(p_3+s_1-p_1)}(\zeta_{3,1}) \quad (3.7)$$

where the newly introduced function  $\mathbf{Q}$  depends besides the parameters  $s_i, p_i$  on the three points  $z_1, z_2, z_3$ . It encodes the tensorial structure of the 3-point vertex function. The number of derivatives of the functions  $w$  in (3.7) are denoted by

$$m_{i,i+1} = p_i + s_{i+1} - p_{i+1} \quad (3.8)$$

A simple formula for  $\mathbf{Q}$  is

$$\begin{aligned} \mathbf{Q}_{p_1, p_2, p_3}^{(s_1, s_2, s_3)} = & \sum_{n_{1,2}=\max\{0, s_2-p_2-p_1\}}^{s_2-p_2} \sum_{n_{2,3}=\max\{0, s_3-p_3-p_2\}}^{s_3-p_3} \sum_{n_{3,1}=\max\{0, s_1-p_1-p_2\}}^{s_1-p_1} \\ & (-1)^{s_1+s_2+s_3} (\Delta)_{m_{12}} (\Delta)_{m_{23}} (\Delta)_{m_{31}} \frac{(s_1-p_1)!(s_2-p_2)!(s_3-p_3)!}{n_{1,2}! n_{2,3}! n_{3,1}!} \\ & \binom{p_1}{s_2-p_2-n_{1,2}} \binom{p_2}{s_3-p_3-n_{2,3}} \binom{p_3}{s_1-p_1-n_{3,1}} \\ & \times [I_1(1, 2)I_2(1, 2)/I_{1,2}]^{n_{1,2}} I_1(1, 2)^{p_1+p_2-s_2} [I_2(2, 3)I_3(2, 3)/I_{2,3}]^{n_{2,3}} I_2(2, 3)^{p_2+p_3-s_3} \\ & \times [I_1(3, 1)I_3(3, 1)/I_{3,1}]^{n_{3,1}} I_3(3, 1)^{p_3+p_1-s_1} I_{1,2}^{s_2-p_2} I_{2,3}^{s_3-p_3} I_{3,1}^{s_1-p_1} \end{aligned} \quad (3.9)$$

Obviously the sums over the parameters  $n_{1,2}, n_{2,3}, n_{3,1}$  can be performed in terms of hypergeometric  ${}_1F_1$  polynomials. The result has a cyclic order

$$\begin{aligned} \mathbf{Q}_{p_1, p_2, p_3}^{(s_1, s_2, s_3)} = & (-1)^{s_1+s_2+s_3} (\Delta)_{m_{12}} (\Delta)_{m_{23}} (\Delta)_{m_{31}} \\ & \times \frac{(s_2-p_2)!}{(s_2-p_1-p_2)!} I_2(1, 2)^{s_2-p_1-p_2} I_{1,2}^{p_1} \\ & \times {}_1F_1(-p_1; s_2-p_1-p_2+1; -\frac{I_1(1, 2)I_2(1, 2)}{I_{1,2}}) \\ & \times \frac{(s_3-p_3)!}{(s_3-p_2-p_3)!} I_3(2, 3)^{s_3-p_2-p_3} I_{2,3}^{p_2} \\ & \times {}_1F_1(-p_2; s_3-p_2-p_3+1; -\frac{I_2(2, 3)I_3(2, 3)}{I_{2,3}}) \\ & \times \frac{(s_1-p_1)!}{(s_1-p_1-p_3)!} I_1(3, 1)^{s_1-p_3-p_1} I_{3,1}^{p_3} \\ & \times {}_1F_1(-p_3; s_1-p_3-p_1+1; -\frac{I_3(3, 1)I_1(3, 1)}{I_{3,1}}) \end{aligned} \quad (3.10)$$

If, say,  $s_2 - p_2 - p_1$  is negative, in the first factor the function  ${}_1F_1$  starts at the term  $[I_1(1, 2)I_2(1, 2)]^{p_1+p_2-s_2}$ , thus replacing essentially the factor  $I_2(1, 2)^{s_2-p_1-p_2}$  in front of  ${}_1F_1$  by  $I_1(1, 2)^{p_1+p_2-s_2}$ . In either expression a zero at  $z_1 - z_2$  of order  $|s_2 - p_1 - p_2|$  is contained. A closer look at the zeros of  $\mathbf{Q}$  will be presented in Section 5.

## 4 The regularization of the $w$ -functions

Remember the definitions

$$w(\zeta) = (\zeta - 1)^{-\Delta}, \quad \Delta = \frac{D}{2} - 1 \quad (4.1)$$

$$u = \zeta - 1 \quad (4.2)$$

$$w^{(n)}(\zeta) = (-1)^n (\Delta)_n (\zeta - 1)^{-\Delta-n} \quad (4.3)$$

and the assumption that  $D$  is even. Moreover we restrict  $D$  to  $\geq 4$ . We intend to use the method of "dimensional regularization" by introducing a parameter  $\epsilon$  interpreted as a deformation of the dimension  $D$  of the  $AdS_D$  space. Then the regularized 3-vertex function appears as a rational function of  $\epsilon$  with a pole of maximal order 2 (for an  $N$ -vertex loop function it would be  $N - 1$ ). We select this pole from our vertex function (3.7) - (3.10) since it delivers us the local differential operator defining the interaction Lagrangian density.

Note that from (3.7), (3.8) we have

$$\sum_{i=1}^3 m_{i,i+1} = s_1 + s_2 + s_3 = S \quad (4.4)$$

which is in all physically reasonable cases not smaller than six. Consequently among the three exponents  $\Delta + m_{ij}$  we find at least one being bigger or equal to two. According to [2] equ. (47) we have

$$\frac{1}{u^{\Delta+n-\epsilon}} = \frac{(-1)^{\Delta+n-1}}{\epsilon(\Delta+n-1)!} \delta^{(\Delta+n-1)}(u) + O(1) \quad (4.5)$$

where  $\epsilon$  is thought of being hidden in  $\Delta = D/2 - 1$  as a deformation of  $D$ . This justifies the term "dimensional" regularization. Moreover we use [2] equ. (75) ( $\Omega_{D-1}$  is the area of the unit sphere in  $D$  dimensions)

$$d\mu(z) = (2z_0)^{-D} d^D z = [u(u+2)]^\Delta du d\Omega_{D-1}, \quad u = \frac{(z_1 - z_2)^2}{2z_1^0 z_2^0} \quad (4.6)$$

$$(2z_1^0)^D \delta(z_1 - z_2) = \frac{(-1)^\Delta \delta^{(\Delta)}(u)}{\Delta!(u+2)^\Delta \Omega_{D-1}} \quad (4.7)$$

Applying the l. h. s. of this equation (4.7) to

$$w^{(m_{23})}(\zeta_{23}) w^{(m_{31})}(\zeta_{31}) \quad (4.8)$$

and turning it into

$$(\zeta_{23} - 1)^{-2\Delta - m_{31} - m_{23}} \quad (4.9)$$

we can apply once again the  $\epsilon$  regularization method.

We conclude from (4.7) that a deltafunction of  $u$  has the minimal derivative  $\Delta$  in order to define a distribution on  $AdS_D$  space. In fact, the moduli of the exponents in  $w^{(m_{12})}(\zeta_{12})$  and in (4.8) are bigger or equal  $\Delta$ , and their excess over  $\Delta$  shall be taken into account by extracting (covariant) Laplacians using a technique also developed in [2]. This is achieved for arbitrary even dimensions  $D \geq 4$  as follows.

We start from

$$r_{12} = \Delta + m_{12} - 1 \quad (4.10)$$

$$r_{23} = 2\Delta + m_{23} + m_{31} - 1 \quad (4.11)$$

$$u_{12}^{-r_{12}-1+\epsilon} = \frac{1}{\epsilon} \frac{(-1)^{r_{12}}}{r_{12}!} \delta^{(r_{12})}(u_{12}) + O(1) \quad (4.12)$$

and an analogous formula for  $u_{23}$  and  $r_{23}$ . Now we introduce the shorthand

$$\Phi_n(u) = \frac{\delta^{(n)}(u)}{(u+2)^\Delta} \quad (4.13)$$

and apply the differential operator  $(u+2)^\Delta \square$  (with the scalar Laplacian)

$$(u+2)^\Delta \square \Phi_n(u) = A_n \delta^{(n+1)}(u) + B_n \delta^{(n)}(u) \quad (4.14)$$

$$A_n = 2(\Delta - n - 1), \quad B_n = n(n+1) - \Delta(\Delta+1) \quad (4.15)$$

This allows us to formulate the recursion

$$\Phi_{n+1}(u) = -D_n \Phi_n(u) \quad (4.16)$$

$$D_n = \frac{1}{2(n+2) - D} [\square + \Delta(\Delta+1) - n(n+1)] \quad (4.17)$$

In the case of the variable  $u_{12}$  (respectively  $u_{23}$ ) we place the pole of the coordinate system at  $z_2$  (respectively  $z_3$ ), define corresponding differential operators  $D_n$  and  $\square$  acting on these pole positions, and denote them correspondingly by  $D_n(2), \square(2)$  (respectively  $D_n(3), \square(3)$ ). Then we get e.g. by solving the recursion (4.16)

$$\Phi_{n+1}(u_{12}) = \frac{(-1)^{n+\Delta}}{\Delta!} \left\{ \prod_{k=\Delta}^{n+\delta-1} D_k(2) \right\} (2z_2^0)^D \delta(z_1 - z_2) \Omega_{D-1} \quad (4.18)$$

Now we return from  $\Phi_n$  to the delta functions

$$\begin{aligned} \delta^{(n+1)}(u_{12}) &= (u_{12}+2)^\Delta \Phi_{n+1}(u_{12}) \\ &= \sum_{\ell=0}^{\Delta} \frac{(-1)^\ell \Delta! 2^{\Delta-\ell} (n+1)!}{\ell! (\Delta-\ell)! (n+1-\ell)!} \Phi_{n+1-\ell}(u_{12}) \end{aligned} \quad (4.19)$$

$$\begin{aligned} &= \sum_{\ell=0}^{\Delta} \frac{(-1)^{n+\Delta} 2^{\Delta-\ell} (n+1)!}{\ell! (\Delta-\ell)! (n+1-\ell)!} \\ &\quad \times \left\{ \prod_{k=\Delta}^{n+\Delta-\ell-1} D_k(2) \right\} (2z_2^0)^D \delta(z_1 - z_2) \end{aligned} \quad (4.20)$$

As we inspect from (4.12) we have to replace here  $n$  by  $r_{12} - 1$  and to multiply with

$$\frac{(-1)^{r_{12}}}{r_{12}!} \quad (4.21)$$

to obtain the distribution part of  $w^{(m_{12})}(\zeta_{12})$ . In an analogous fashion we treat the distribution

$$\delta^{(r_{23})}(u_{23}) \quad (4.22)$$

## 5 Discussion

By partial integration the polynomials of the Laplacian in (4.24) can be brought to the left side, then acting to the right, so that to the right the factor  $\mathbf{Q}$  follows and finally the higher spin fields. The result is

$$\begin{aligned} & \int \frac{d^D z}{(z^0)^D} \sum_{p_1=0}^{s_1} \sum_{p_2=0}^{s_2} \sum_{p_3=0}^{s_3} A_{p_1}^{(s_1)} A_{p_2}^{(s_2)} A_{p_3}^{(s_3)} (\Delta)_{m_{12}} (\Delta)_{m_{23}} (\Delta)_{m_{31}} (\Omega_{D-1})^2 \\ & \sum_{\ell_1=0}^{\Delta} \sum_{\ell_2=0}^{\Delta} \frac{2^{2\Delta-\ell_1-\ell_2}}{\ell_1! \ell_2! (\Delta - \ell_1)! (\Delta - \ell_2)! (\Delta + m_{12} - \ell_1 - 1)! (2\Delta + m_{23} + m_{31} - \ell_2 - 1)!} \\ & \quad \times \Lambda_{\Delta+m_{12}-\ell_1-1}(\square(2), \Delta) \Lambda_{2\Delta+m_{23}+m_{31}-\ell_2-1}(\square(3), \Delta) \\ & \quad \times \mathbf{Q}_{p_1, p_2, p_3}^{(s_1, s_2, s_3)} *_{a_1} *_{a_2} *_{a_3} h^{(s_1)}(z, a_1) h^{(s_2)}(z_2, a_2) h^{(s_3)}(z_3, a_3) \big|_{z_2=z_3=z} \end{aligned} \quad (5.1)$$

where the shorthand (see (3.8))

$$m_{i,i+1} = p_i + s_{i+1} - p_{i+1} \quad (5.2)$$

has been used. The asterisk symbols denote contractions that produce a scalar function of  $\mathbf{Q}$  and  $h^{(s_1)} h^{(s_2)} h^{(s_3)}$ .

The Laplacians act only on the variables  $z_2$  and  $z_3$ . This is the effect of a partial integration eliminating all differentiations with respect to  $z_1$ . Other approaches to these vertex functions may give results symmetric in the three variables  $z_1, z_2, z_3$ , which makes any comparison with our result troublesome. In any case differentiations on the factors contained in  $\mathbf{Q}$  in (3.9) or (3.10) seem to necessitate an algorithmic computer program.

The maximal number of differentiations is

$$2(r_{12} + r_{23} - 2) = 3D + 2S - 4 \quad (5.3)$$

However, in  $\mathbf{Q}$  there are zeros which have to be cancelled first before the differentiations act on the fields. These zeros are hidden in  $I_i(i, i+1), I_{i+1}(i, i+1)$  and are each of order one. Thus the total number of zeros in  $\mathbf{Q}$  is

$$\Psi = |s_1 - p_1 - p_3| + |s_2 - p_2 - p_1| + |s_3 - p_3 - p_2| \quad (5.4)$$



Now assume that the three numbers  $s_1, s_2, s_3$  satisfy triangular inequalities, then we can solve  $\Psi = 0$  by

$$p_1 = \frac{1}{2}(s_1 + s_2 - s_3), \quad p_2 = \frac{1}{2}(s_2 + s_3 - s_1), \quad p_3 = \frac{1}{2}(s_3 + s_1 - s_2) \quad (5.5)$$

In this case the maximal number of derivatives acting on the fields can be the one given by (5.3).

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